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TRANSIENT FLOW REDISTRIBUTION IN ANNULAR TWO-PHASE FLOW

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Abstract—A model is described for the prediction of transient flow redistribution in vertical annular two-phase flow. The model is based on an analysis of the local parameters controlling the flow and takes account of the diffusive motion of entrained droplets and the delay time for change in the wave structure on the film. Comparisons are made with experimental results on inlet effects and it is shown that the wall injection experimental results can be described by the model. The jet injection results are not fitted by the model and it is shown that some additional deposition mechanism must be present.

1. INTRODUCTION

Annular two-phase flow is an important regime occurring in many types of reactor heat transfer equipment. The flow regime is characterized by the existence of a liquid film of the channel walls and a gas flow in the core, which may carry entrained liquid droplets. In essentially all applications of interest the gas flow is highly turbulent.

A great deal of effort has been applied to obtain suitable model descriptions and experimental results for the mechanisms of heat and mass transfer which occur in annular flow. However, as there is no satisfactory first principles description of single phase turbulent flow it is not surprising that models of annular two-phase flow have, so far, met with limited success. Up to the present the most successful methods for prediction of the properties of annular two-phase flows, have been based on correlations of data for a particular fluid, see for example Hewitt & Hall Taylor (1970); Collier (1972). If results are required for some arbitrary fluid, recourse to experimental measurement is usually necessary. The prediction of the mass distribution between the film and core, in a channel with varying flow conditions has, until now, proved impossible.

We describe here a model for the prediction of transient flow redistribution in isothermal vertical annular flow. The model is based on an analysis of the local parameters controlling the flow, and has also been applied to the calculation of flow redistribution in the presence of heat transfer, and, hence, the prediction of burnout by Whalley *et al.* (1973).

The liquid film flow can be represented by a boundary layer theory which results in the familiar "triangular relationship" between the liquid film thickness, pressure drop and

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film flow rate. A review of the various forms proposed for this relationship is given by Hewitt & Hall Taylor (1970). The gas flow may be characterized in terms of a friction factor. The friction factor is obtained from the geometrical similarity relationship which gives the effective roughness, presented by the interface to the gas, in terms of the liquid film thickness. Again, Hewitt & Hall Taylor (1970) review various types of interfacial roughness correlation.

However, the above description is inadequate for a closed form solution for the mass flows. The missing relationships are those describing droplet mass transfer between the core of the annular flow and the liquid film.

Discussion of the droplet mass transfer process requires a description of both the mechanism for the deposition of entrained droplets from the gas flow onto the liquid film, and that by which droplets are entrained from waves on the liquid film, and join the gas flow in the core. A theoretical model for the deposition process has been developed in terms of the droplet interaction with the turbulent gas flow by Hutchinson *et al.* (1971). The predictions of the deposition model were found to be in good agreement with experimental measurements of deposition rate made by Cousins & Hewitt (1968) and others. More recently a correlation has been constructed by Hutchinson & Whalley (1973) which successfully characterizes the rate of droplet entrainment from the liquid film to the gas stream, in terms of the interfacial shear stress, film thickness and liquid surface tension.

The above description of the droplet mass transfer process, in conjunction with the equations for the gas and film flows are sufficient to allow a solution of a mass balance equation. This solution describes the distribution of liquid between the wall and gas core, as a function of distance along the tube for arbitrary initial conditions.

Two models are discussed in detail in Sections 2 and 3 below, and compared with the experimental results of Gill & Hewitt (1968). The first, simpler, treatment assumes that after entrainment the particles are instantaneously mixed with the gas across the tube, so that the mass transfer coefficient k is independent of downstream position z. It is shown that this model is inadequate to describe transient behaviour, despite its reliability for the prediction of equilibrium properties. The second model takes account of the diffusive transport of droplets in the gas core, and the consequent variation of mass transfer coefficients with axial position. The relaxation length for diffusive mixing in the gas core is evaluated and shown to be of order of hundreds of tube diameters. An account is also given of the effect of the relaxation time for the wave pattern to adjust to changes in film thickness.

Finally conclusions are drawn from the comparisons between theory and experiment.

2. SIMPLE MODEL

2.1 *Theory*

We describe here a method of calculating the liquid film thickness m, the interfacial shear stress τ_i, and the mass flux of entrained liquid droplets G_{LE} as a function of axial position z. It is assumed that the gas and total liquid mass fluxes G_G and G_L, gas and liquid densities ρ_G and ρ_L and viscosities μ_G and μ_L, liquid surface tension σ and tube diameter d_o are known.

Once the liquid film thickness, interfacial shear stress and mass flux of entrained liquid droplets have been evaluated, other variables of interest, e.g. pressure drop, can be calculated.

The development of the mass distribution of the flow as a function of axial position can be described in terms of the local entrained liquid flow rate by the isothermal mass balance equation:

$$\mathrm{d}G_{LE}/\mathrm{d}z = 4(E - D)/d_o, \qquad [1]$$

where G_{LE} is the mass flow of entrained liquid droplets per unit of cross sectional area of the channel, and D and E respectively are the rates of deposition and entrainment per unit area of the tube wall.

With the assumption of instantaneous mixing of the entrained particles across the tube D is given, at each point along the tube, by the equation:

$$D = kC, \qquad [2]$$

where k is a mass transfer coefficient independent of position, and C is the local mean concentration of droplets entrained in the gas flow.

The entrainment rate is obtained from a numerical interpolation of the correlation of Hutchinson & Whalley (1973) which gives the mean concentration of droplets in the gas core at hydrodynamic equilibrium C_E as a function of the dimensionless group $\tau_i m/\sigma$. The entrainment rate E is related to C_E by:

$$E = kC_E. \qquad [3]$$

Equations for the interfacial shear stress τ_i and local film thickness, m which appear in the entrainment correlation are necessary to complete the model. The film thickness is related to the liquid film flow and pressure drop by the "triangular equation" in a simplified form given by Turner & Wallis (1965) based on a relation originally developed by Armand (1946). This relationship is

$$\frac{4m}{d_o} = \sqrt{\frac{(\mathrm{d}P/\mathrm{d}z)_{LF}}{(\mathrm{d}P/\mathrm{d}z)}}. \qquad [4]$$

where $(\mathrm{d}P/\mathrm{d}z)_{LF}$ is the pressure gradient which would occur in a single phase liquid flow with a mass flow identical with that of the film, and $\mathrm{d}P/\mathrm{d}z$ is the pressure gradient in the two-phase flow. $(\mathrm{d}P/\mathrm{d}z)_{LF}$ was calculated from the liquid film friction factor which is plotted against Reynolds number by Hewitt & Hall Taylor (1970) from numerical data of Hewitt (1961).

Finally, the pressure drop is obtained from the geometrical similarity condition, which relates the liquid film thickness to the effective roughness presented to the gas core by the gas–liquid interface. A linear form of this relation due to Wallis (1970) was used, in which the interfacial friction factor (defined in terms of a homogeneous gas core) can be expressed as $(1 + 360m/d_o)$ times the friction factor for the gas core flowing in the absence of the liquid film.

By the use of the above relationships, the mass balance equation may be integrated numerically from any set of initial conditions, to give the mean mass distribution in the tube as a function of axial position.

2.2 *Comparison with experiment*

Gill & Hewitt (1968) have reported measurements of entrainment flow rate as a function of axial distance for two extreme sets of entrance conditions:

(i) Porous sinter. A section of the tube wall was constructed of porous sintered material through which liquid entered. Ideally it may be expected that initially all the liquid is in the film.

(ii) Axial jet. The liquid entered through a small jet on the axis of the tube. In this case it is reasonable to assume that initially all the liquid is entrained.

The experiments were carried out in a vertical tube of 31.8 mm dia, using air and water flows at mass fluxes of 79.25 and 158.5 kg/m^2s, respectively. The experimental results are shown in figures 1 and 2. The air and water viscosities, the water density and the air–water surface tension were 0.0000175 Ns/m^2, 0.00114 Ns/m^2, 1000 kg/m^3 and 0.073 N/m, respectively. At the injection point the gas density was 2.22 kg/m^3 and the pressure was $0.186 \times 10^6\,N/m^2$ absolute.

In figure 1 we also show the results of calculations based on the simple model described in Section 2.1 above. The mass transfer coefficient used was 0.15 m/s, which is representative of the values obtained in the experimental work by Cousins & Hewitt (1968). The initial conditions were taken as 99.5 and 0.5 per cent respectively of the liquid flow to be in the film. The other variables were fixed at the values used in the experimental work.

The overall trend predicted by the model is compatible with the experimental results and it can be seen that the equilibrium prediction is reasonably accurate. However the

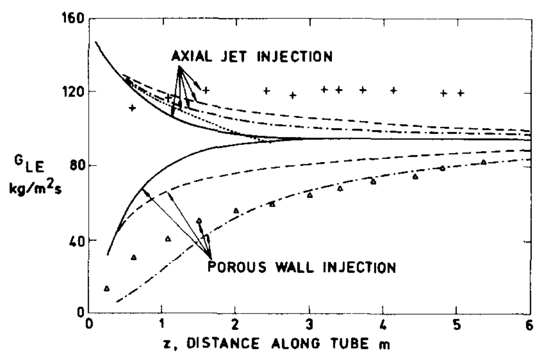

Figure 1. Experimental and calculated results for entrained liquid mass flux (gas density constant). + + + △ △ △ Experimental data of Gill & Hewitt (1968), ——— Deposition given by simple theory [2], ---- Deposition given by diffusive theory (appendix), —·—·— Deposition given by diffusive theory, and entrainment given by [5], ······ Deposition given by diffusive theory, no entrainment at all.

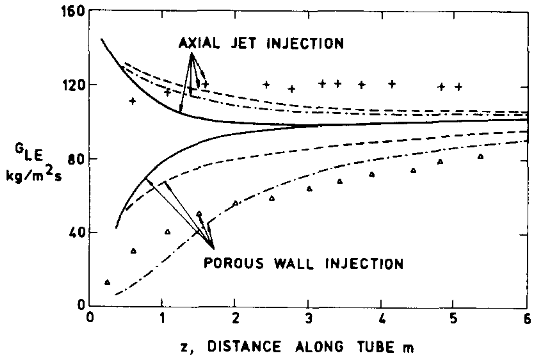

Figure 2. Experimental and calculated results for entrained liquid mass flux (gas density varying). + + + △ △ △ Experimental data of Gill & Hewitt (1968), ——— Deposition given by simple theory [2], – – – – Deposition given by diffusive theory (appendix), —·—·— Deposition given by diffusive theory, and entrainment given by [5].

equilibrium conditions are achieved much too rapidly, and the model does not exhibit the overshoot in deposition shown by the jet injection experiment. In figure 2 we show the effect of incorporating into the model the variation of gas density as a result of the pressure variation along the tube. It may be seen that this correction has only a slight effect. It thus follows that the simple model described here has omitted to account properly for important relaxation processes in the flow and further refinement is necessary for the prediction of transient behaviour.

3. DIFFUSION MODEL

3.1 *Theory*

In this section we describe the modification of the simple model discussed in Section 2 above to include an account of the diffusive mixing of entrained particles with the gas flow, and the effect of a delay between changes in the film thickness and the corresponding change in wave structure.

It has been shown by Hutchinson *et al.* (1971) that, in the absence of re-entrainment, a good account of the variation of deposition rate with axial position is provided by the assumption that the entrained droplets diffuse through the gas flow. The diffusion coefficient λ for the transport process is fixed by the dynamics of the interaction between the turbulent gas flow and the droplets. It follows from such a theory that in a developing flow, the concentration profile of droplets across the tube is a function of axial position. Hence the mass transfer coefficient in [2] may no longer be assumed constant. It is appropriate to neglect diffusive motion in the axial direction as the rate of droplet transport is dominated by the mean velocity of the gas. In the earlier work of Hutchinson *et al.* (1971) it was assumed that the gas velocity was independent of both radial and axial position. The assumption of independence of gas velocity on radial position is preserved. However, it is inappropriate here to neglect the variation in axial velocity which arises as a result of the combination of conservation of momentum and the significant variation in the mean density of the core with change in the mass of entrained droplets. It follows that, to take account of an axially varying mean gas velocity, the deposition rate must be evaluated as the solution of a two dimensional diffusion equation for the axial mass flux of droplets. The entrainment process is represented as a time varying ring source of particles located at the instantaneous wave height. It has been shown by Hewitt (1969) that the mean wave height was approximately five times the average film thickness. In the case of axial jet injection, the initial entrainment is represented by a uniform distribution of droplets from the tube centre to the wave crests. The boundary conditions are fixed by the assumption that all droplets which impinge on the film are captured. With these assumptions the mass balance [1] may be formally integrated to give the axial mass flux of entrained droplets G_{LE} in terms of the entrainment rate history. Application of [2]–[4] and the interfacial roughness correlation then allows an iterative numerical solution for G_{LE}. The derivation of the diffusion equation and solution procedure are described in Appendix 1.

From [A7] it follows that the shortest relaxation time T_λ for mixing of droplets across the tube is of order $d_o^2/4\lambda\gamma_o^2$ and hence the axial distance covered by the gas L_λ is $\bar{U}d_o^2/4\lambda\gamma_o^2$ where $\bar{U}$ is the velocity of the mixture of gas and droplets. The diffusion constant λ was determined from [A13] relating the equilibrium mass transfer coefficient and the position of the ring source to the diffusion coefficient, to give for λ a value of 1.73×10^{-4} m^2/s. It should be noted that λ and k are dominated by the droplet size, and this choice of λ is roughly equivalent to a choice of a droplet size of order 100 μm. Substitution of the numerical values shows that L_λ is of order hundreds of tube diameters, and that hence the mixing process is show by comparison with axial transport.

3.2 *Comparison with experiment*

Figures 1 and 2 show comparisons of the results of calculations based on the diffusive model with the results of Gill & Hewitt (1968) outlined above. The calculations in figure 2 incorporate the effect of the pressure variation on the gas density. It is clear that the addition of diffusive effects produces a substantial increase in the time to achieve equilibrium. Again no indication is given by the theory of the overshoot in deposition shown in the jet injection experiment. In addition the increased time for establishment of equilibrium arising from diffusion is still inadequate to account for the experimental results, especially in the region near the entrance.

From observations by Hall Taylor (1967); Hall Taylor & Nedderman (1968) on the change in wave frequency upstream of the injection point, it may be concluded that the interfacial wave structure lags behind variation in the mean film thickness by distances of order one metre. Such an effect would be important in the entrance region where the mean film thickness varies rapidly with axial position. To take some account of this effect the entrainment rate from any location was suppressed by a delay factor of the form:

$$E = E_o(1 - e^{-z/L_o}), \qquad [5]$$

where E_o is the entrainment predicted by the correlation and L_o was fixed by an approximate fit at small z to be 2m. While this procedure is probably too crude it has the correct form in that it is a local effect.

The results of incorporating [5] into the model are shown in figures 1 and 2. The wall injection experiment is now reasonably well described by the model. Again the effect of variation in gas density with pressure is not large by comparison with the other effects present. However the jet injection results still show a poor fit. In figure 1 we also show a calculation in which entrainment is completely omitted. This curve provides a lower bound for G_{LE} as predicted by the model. It is thus reasonable to conclude that some additional effect is enhancing deposition with axial jet injection experiments.

4. CONCLUSIONS

A simple model of annular two-phase flow adequate for prediction of "equilibrium" properties, has been shown inaccurate in its description of transient flows. A more refined model incorporating diffusive transport of particles in the gas core, and a delay length for wave formation is capable of describing the wall injection transient flow measurements of Gill & Hewitt. To improve agreement further refinement is necessary of the description of entrainment suppression arising from delays in wave formation.

The present model cannot adequately describe the jet injection experiments. The deposition rate near the entrance is much higher than that predicted by the model used here. Such an enhancement of deposition may be caused by mechanisms such as jet spread, or swirl induced in the gas flow by an obstruction upstream of the jet nozzle. The latter effect is consistent with the observation that the jet injector used by Gill & Hewitt was supported sufficiently close to the nozzle to allow swirl to be present in the gas flow at the liquid entrance region.*

While a detailed evaluation of the validity of the diffusion model presented here is limited by the absence of experimental data, it is important to note that the value chosen for r_o is that found by experiment (Hewitt 1969), and that obtained for L_o is consistent with the measurements of Hall Taylor (1967). No account has been given of the effect of variation in entrainment position, r_e, over the waves and the consequent change in the diffusion coefficient, given by [A13]. However, while the diffusion coefficient is sensitive to the choice of r_e, calculations based on an entrainment position at half the maximum wave height show that the magnitude of the change in the entrained liquid mass flux is of the same order as that due to neglect of gas density variation. It follows that the fitted value of L_o is similarly insensitive to the choice of r_e.

A more critical evaluation of the model will only be possible when more experimental data is available, and the mechanism of droplet entrainment is more clearly understood.

* We are grateful to a reviewer for drawing our attention to the possible presence of this mechanism.

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APPENDIX

(i) *Derivation of the diffusion equation*

Consideration of mass conservation over any volume element in the flow in the gas core gives:

$$\int_{\text{volume}} \left(\frac{\partial}{\partial z}(\overline{U}C(r,z) - S(r,z) + \frac{\partial}{\partial t}C(r,z)\right) \mathrm{d}V - \int_{\text{surface}} \lambda \underline{\nabla}_r C(r,z)\, \mathrm{d}\phi = 0, \qquad [\text{A1}]$$

where $\overline{U}$ is the axial velocity, $S(r, z)$ is a source term, $C(r, z)$ is the local mean concentration of particles, dV is a volume element, dϕ an element of surface area and $\underline{\nabla}_r$ implies omission of derivatives with respect to the axial location, z.

The first term in [A1] gives the effect of dilation arising from varying axial velocity, and the final term accounts for the loss of particles by diffusion in directions perpendicular to the direction of gas flow.

Application of Green's theorem to [A1] and elimination of the volume integral gives:

$$\frac{\partial}{\partial z}(\bar{U}(z)C(r, z)) - S(r, z) + \frac{\partial}{\partial t}C(r, z) - \lambda\underline{\nabla}_r^2 C(r, z) = 0. \quad \text{[A2]}$$

We again neglect diffusive motion in the axial direction.

We assume that at any point along the tube mean properties of the flow are time independent, i.e. start up effects are neglected, then:

$$\frac{\partial}{\partial t}C(r, z) = 0. \quad \text{[A3]}$$

Equation [A2] may then be written:

$$\bar{U}(z)\frac{\partial}{\partial z}(\bar{U}(z)C(r, z)) - \bar{U}(z)S(r, z) - \lambda\underline{\nabla}_r^2(\bar{U}(z)C(r, z)) = 0. \quad \text{[A4]}$$

It is assumed that the particles move with the same axial velocity as the gas, hence $\bar{U}(z)$ may be identified as the axial gas velocity. Transformation to a system of co-ordinates moving with the axial velocity of the gas implies that $\bar{U}(z) = \mathrm{d}z/\mathrm{d}t$. Combination of this relation and [A4] gives the equation for the variation of particle concentration in a co-ordinate system moving with the gas as:

$$\frac{\partial}{\partial t}(\bar{U}(t)C(r, t)) - \bar{U}(t)S(r, t) - \lambda\underline{\nabla}_r^2(\bar{U}(t)C(r, t)) = 0. \quad \text{[A5]}$$

Equation [A5] has the form of a two dimensional diffusion equation for the axial mass flux of particles.

(ii) *Solution of the diffusion equation*

The diffusion equation [A5] in company with a specification of the boundary conditions, and a definition of the source term completely defines the problem.

The source term $S(r, t)$ is chosen to be of the form:

$$S(r, t) = \frac{d_o}{2r_e}E(t)\delta(r - r_e) + \frac{d_o^2}{4r_p^2}\bar{C}_p U(r_p - r)\delta(t), \quad \text{[A6]}$$

where $U(r_p - r) = 1$ if $r \leqslant r_p$ and $U(r_p - r) = 0$ if $r > r_o$.

r_e is the instantaneous radial position of the wave height for entrainment, $\delta(r)$ is the Dirac delta function, $E(t)$ is the instantaneous entrainment rate per unit area of tube wall, $\bar{C}_p$ is the mean concentration over the tube of a plug source of radius r_p, present at the tube entrance, i.e. $t = 0$. In [A6] the delta function term represents the continuous time varying source arising from re-entrainment of droplets from the film, and the factor $d_o/2r_e$ appears as $E(t)$ is given per unit area of tube surface. The step function term is used to represent the presence of particles in the gas core in jet injection, and the factor $d_o^2/4r_p^2$ arises as $\bar{C}_p$ is taken as an average over the entire tube cross section. In the calculation r_p is taken to be $r_e(o)$.

The boundary conditions on $C(r, t)$ are fixed by the assumption of complete absorption of all particles which strike the film, i.e. in a cylindrical tube $C(d_o/2, t) = 0$. Here the finite depth of the film has been neglected as negligible in comparison with the tube diameter. Such an assumption is valid for most cases of practical interest.

Equations [A5] and [A6] may now be solved in terms of the Green's function given in Carslaw & Jaeger (1959) to give:

$$C(r, t) = \frac{2}{\bar{U}(t)a} \sum_{N \geq 1} \frac{J_o(r\gamma_N/a)J_o(r_e\gamma_N/a)}{J_1^2(\gamma_N)} \int_o^t \bar{U}(t')E(t')\,\mathrm{e}^{-\lambda\gamma_N^2(t-t')/a^2}\,\mathrm{d}t'$$
$$+ \frac{2a\bar{C}_p\bar{U}(0)}{r_p\bar{U}(t)} \sum_{N \geq 1} \frac{J_o(r\gamma_N/a)J_1(r_p\gamma_N/a)}{\gamma_N J_1^2(\gamma_N)}\,\mathrm{e}^{-\lambda\gamma_N^2 t/a^2}, \qquad \text{[A7]}$$

where $J_o(\gamma_N) = 0$ and $a = d_o/2$. The deposition rate is given by:

$$D = -\frac{\lambda\,\partial C(r, t)}{\partial r}\bigg|_{r=a}. \qquad \text{[A8]}$$

The mass flux G_{LE} may be obtained from:

$$G_{LE} = \bar{U}(t)\bar{C}(t), \qquad \text{[A9]}$$

where $\bar{C}(t)$ is the mean droplet density over the tube, i.e.

$$\bar{C}(t) = \frac{2}{a^2}\int_o^a rC(r, t)\,\mathrm{d}r. \qquad \text{[A10]}$$

Evaluation of $\bar{C}(t)$ from [A7] results in:

$$\bar{C}(t) = \frac{4}{a\bar{U}(t)} \sum_{N \geq 1} \frac{J_o(r_e\gamma_N/a)}{\gamma_N J_1(\gamma_N)} \int_o^t \bar{U}(t')E(t')\,\mathrm{e}^{-\lambda\gamma_N^2(t-t')/a^2}\,\mathrm{d}t'$$
$$+ \frac{4a\bar{C}_p\bar{U}(o)}{r_p\bar{U}(t)} \sum_{N \geq 1} \frac{J_1(r_p\gamma_N/a)}{\gamma_N^2 J_1(\gamma_N)}\,\mathrm{e}^{-\lambda\gamma_N^2 t/a^2}. \qquad \text{[A11]}$$

The axial position, z, is related to the time t by $z = \int_o^t \bar{U}(t)\,\mathrm{d}t$. Differentiation of [A11] with respect to time, and use of [A8] confirms that [1] is recovered.

$\bar{C}(t)$ was evaluated by first computing a spline fit to the history $E(t')$ appearing in the integrand of [A11] for an initial estimate of $E(t)$. The summation over the roots of the Bessel function is carried out before integration of the spline, so as to obtain an accurate representation of the essential singularity in the function at short times separately from the smooth behaviour of $E(t')$. The estimate $E(t)$ is then corrected from the equations governing the gas and film flows and the iteration continued to convergence. Computational accuracy is confirmed to 1 per cent by comparison of the deposition rate as evaluated numerically by differencing $\bar{C}(t)$ and as given by [A8].

At long times when $E(t)$ is effectively constant, [A11] reduces to:

$$\bar{C} = \frac{Ea}{2\lambda}\left(1 - \frac{r_e^2}{a^2}\right). \quad [A12]$$

Equation [A8] then takes on its equilibrium form, $D = E$; the asymptotic mass transfer coefficient is thus given by:

$$k = \frac{2\lambda}{a} \bigg/ \left(1 - \frac{r_e^2}{a^2}\right). \quad [A13]$$

Sommaire—On décrit un modéle permettant la prediction de la redistribution de l'ecoulement transitoire dans un écoulement diphasique annulaire. Ce modéle est basé sur une analyse des paramètres locaux contrôlant l'écoulement et prend en consideration le mouvement de diffusion des gouttes entraînées et le temps de retard dans la structure d'onde sur le film. Des comparaisons sont faites avec des résultats expérimentaux sur des effets d'entrée et il est montré que les resultats expérimentaux dans le cas d'une injection par la paroi peuvent etre decrits par le modéle. Les résultats pour une injection par jet ne s'accordent par avec le modéle et il est montré qu un mécanisme de dépôt supplementaire doit être présent.

Auszug—Es wird ein Modell fuer die Voraussage der Uebergangsrueckverteilung der vertikalen Zweiphasenstroemung im Ringspalt beschrieben. Das Modell basiert auf einer Analyse der oertlichen, die Stroemung bestimmenden Parameter, und beruecksichtigt die diffusive Bewegung mitgefuehrter Tropfen und die Verzoegerungszeit fuer die Aenderung der Wellenstrukter auf dem Film. Vergleiche mit Versuchsergebnissen ueber Einlaufseffekte werden angestellt, und es wird gezeigt, dass die Versuchsergebnisse mit Wandinjektion durch das Modell beschreibbar sind. Die Versuche mit Strahlinjektion entsprechen dagegen dem Modell nicht, und es wird gezeigt, dass ein zusaetzlicher Ablagerungsmechanismus wirksam sein muss.

Резюме—Описана модель для определения переменного перераспределения потока в вертикальном кольцевом двухфазном потоке. Модель основана на анализе местных параметров, контролирующих поток, и учитывает диффузное движение погрушенных капелек и запаздывание для замены (перемены) в волновой структуре на пленке. Проводящая сравнения с экспериментальными результатами на входные силы и показано, что экспериментальные результаты стенной инжекции могут быть описаны моделю. Результаты контактной инжекции (инжекции смещением) не определяются моделью и показано, что необходимо присутствие какого-нибудь дополнительного механизма укладки.